# Can noncommutativity affect the whole history of the Universe?

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A noncommutative Friedmann-Robertson-Walker (NC-FRW) cosmological Universe is proposed. We prove that, at the classical level, the noncommutativity removes the future cosmological singularities originally present in the commutative version of the model. We show that noncommutativity affects the entire history of the Universe in an unsuspected way: for a closed FRW model with negative noncommutative (NC) parameter the evolution of the Universe has periodic acceleration with alternating sign; the NC parameter thus takes qualitative account of the early inflationary phase and a late regime of accelerated expansion, with an in-between period of decelerated expansion.

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A decade ago it was possible to conclude from observational data that the Universe is going through a phase of accelerated expansion[1]. After that, the main hypothesis in the literature that tries to explain this fascinating feature is that the Universe is uniformly filled with an exotic fluid (the dark energy), which would drive the accelerated expansion of the Universe under an equation of state $P/\rho = w < -1/3$. In this context, the cosmological constant is an important candidate to represent the dark energy, since it acts as a source [2] with $p/\rho = -1$, which is consistent with observational results. However, the theoretical value for the cosmological constant (estimated by the physics of high energy particles) conflicts with observational data by 30 orders of magnitude in the energy scale[3]. Despite of this, the role played by the cosmological constant at the FRW cosmological model (obtained via Schutz formalism) has been investigated at the quantum level[4] and interesting results have been obtained: the positive cosmological constant can take account of the initial inflationary period and also of a late accelerated expansion phase, with an in-between decelerated expansion phase; also, the past and future cosmological singularities are removed. Another common proposal for this issue in Cosmology that of employing quantized scalar fields (*inflatons*)[5].

Recently, the role played by the noncommutativity has been extensively investigated in cosmological models[6] and in this scenario it was possible to prove that the past and future cosmological singularities are removed by the NC parameter[7] and that the NC effects are relevant to the entire history of the Universe for intermediary times[8]. It is important to mention two important works in which the authors also try to describe the present accelerated expansion of our Universe using noncommutativity [9, 10], by demonstrating that the NC parameter does affect the FRW cosmological model, since the effective self-interaction potential of the scalar field is modified with the introduction of the NC parameter; consequently, the initial inflationary period and the late accelerated expansion phase are parametrized by the NC parameter.

In the present work the NC-FRW cosmological model, restricted to the radiation case without cosmological constant, is proposed *via* Schutz formalism [11], having been obtained through an alternative approach to the Seinberg-Witten map[12], the noncommutative symplec-

tic method[13]. The NC-FRW cosmological model is then investigated at its classical level,*i.e.*, the Hamilton equations of motion are obtained and, subsequently, their solutions will be calculated and studied, allowing us to shed some light into the intriguing behaviour of our Universe.

In order to introduce the NC parameter into the FRW cosmological model, *via* the noncommutative symplectic method[13], we propose the following non-vanishing NC Dirac brackets:

$$\{a, P_a\} = \{T, P_T\} = \{\zeta, N\} = 1,$$
$$\{a, P_T\} = \{P_a, T\} = \gamma, \tag{1}$$

in which $\gamma$ is the NC parameter. It is important to notice that this is not the only possible NC arrangement. Since the Dirac brackets among the variables are identified as the entries of the inverse symplectic matrix[14] $f^{-1}$, namely,

$$f^{-1} = \begin{pmatrix} 0 & 1 & 0 & \gamma & 0 & 0 \\ -1 & 0 & \gamma & 0 & 0 & 0 \\ 0 & -\gamma & 0 & 1 & 0 & 0 \\ -\gamma & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}. \tag{2}$$

in which the symplectic variables are $\chi_\alpha = (a, P_a, T, P_T, N, \zeta)$. As $f^{-1} \cdot f = \mathbf{1}$, after a straightforward computation, the symplectic matrix $f$ is obtained,

$$f^{-1} = \begin{pmatrix} 0 & -\frac{1}{(1+\gamma^2)} & 0 & -\frac{\gamma}{(1+\gamma^2)} & 0 & 0 \\ \frac{1}{(1+\gamma^2)} & 0 & -\frac{\gamma}{(1+\gamma^2)} & 0 & 0 & 0 \\ 0 & \frac{\gamma}{(1+\gamma^2)} & 0 & -\frac{1}{(1+\gamma^2)} & 0 & 0 \\ \frac{\gamma}{(1+\gamma^2)} & 0 & \frac{1}{(1+\gamma^2)} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}. \tag{3}$$

The respective first-order Lagrangian density in the NC framework is given by

$$\mathcal{L} = \left(\frac{P_a + P_T \cdot \gamma}{1+\gamma^2}\right) \cdot \dot{a} + \left(\frac{P_T - P_a \cdot \gamma}{1+\gamma^2}\right) \cdot \dot{T} + (N - a) \cdot \dot{\zeta} - \tilde{\mathcal{H}}, \tag{4}$$

in which $(N - a)$ is the gauge-fixing term and $\tilde{\mathcal{H}}$ is the symplectic potential (usualy assumed to be the Hamiltonian). In the commutative framework ($\gamma = 0$) the first-order Lagrangian reads

$$\mathcal{L} = P_a \cdot \dot{a} + P_T \cdot \dot{T} + (N - a) \cdot \dot{\zeta} - \mathcal{H}, \tag{5}$$

in which $\tilde{\mathcal{H}} \to \mathcal{H}$ and

$$\mathcal{H} = -\frac{P_a^2}{24} - 6ka^2 + P_T. \tag{6}$$

In the NC framework, the variable transformations are

$$\begin{aligned} a &\to a, \\ P_a &\to \frac{P_a + P_T \cdot \gamma}{1+\gamma^2}, \\ T &\to T, \\ P_T &\to \frac{P_T - P_a \cdot \gamma}{1+\gamma^2}, \\ \zeta &\to \zeta. \end{aligned} \tag{7}$$

Therefore, the Hamiltonian in the NC framework is given by

$$\tilde{\mathcal{H}} = -\frac{(P_a + P_T \cdot \gamma)^2}{24(1+\gamma^2)} - 6ka^2 + \frac{P_T - P_a \cdot \gamma}{(1+\gamma^2)}. \tag{8}$$

The Hamilton equations of motion, obtained using the NC Dirac brackets in Eq.(1) and the Hamiltonian in Eq.(8), are

$$\begin{aligned} \dot{a} &= \left\{a, \tilde{\mathcal{H}}\right\} = -\frac{P_a + P_T \cdot \gamma}{12(1+\gamma^2)}, \\ \dot{P}_a &= \left\{P_a, \tilde{\mathcal{H}}\right\} = 12ka, \\ \dot{T} &= \left\{T, \tilde{\mathcal{H}}\right\} = 1 \\ \dot{P}_T &= \left\{P_T, \tilde{\mathcal{H}}\right\} = 12ka\gamma. \end{aligned} \tag{9}$$

These time-derivative equations yield the following differential equation for the scale factor:

$$(\dot{a} - \gamma)\ddot{a} + ka(\dot{a} - \gamma) = 0. \tag{10}$$

By introducing the transformation

$$\tilde{a} \to a - \gamma t, \tag{11}$$

Eq.(10) now becomes

$$\dot{\tilde{a}}(\ddot{\tilde{a}} + k\tilde{a} + k\gamma t) = 0. \tag{12}$$

From the latter we obtain: a trivial differential equation, the solution of which is $\tilde{a}(t) = C + \gamma t$, in which $C$ is a constant; and also the second-order differential equation,

$$\ddot{\tilde{a}} + k\tilde{a} + k\gamma t = 0. \tag{13}$$

This is the equation of a driven harmonic oscillator, under the driving force $-k\gamma t$. At this point it is important to notice that if $\gamma = 0$ the commutative FRW cosmological model is restored; consequently, the past and future singularities are also restored. In what follows, we will solve Eq.(13) analytically and subsequently explore some features of its solution, for several values of the NC parameter.

The following analysis of the solutions of Eq. (13) is carried out for the initial conditions $\tilde{a}(0) = a_0,\ \dot{\tilde{a}}(0) = 0$. For models with $k = -1$ and $\gamma < 0$, the solutions of Eq.(13) have the form

$$\tilde{a}(t) = a_0 \cosh(t) - |\gamma| \left(\sinh(t) - t\right). \tag{14}$$

According to (14), for the commutative model ($\gamma = 0$) the Universe expands indefinitely with accelerated ratio, which yields (i) problems regarding the structure formation and (ii) a *big-rip* ending for the Universe. With the introduction of the NC parameter, it is observed that the larger the parameter the lower the expansion ratio of the scale factor. The acceleration of the scale factor is positive for $t \in \left(0, \frac{1}{2}\ln\left(\frac{|\gamma|+a_0}{|\gamma|-a_0}\right)\right)$, and negative for $t \in \left(\frac{1}{2}\ln\left(\frac{|\gamma|+a_0}{|\gamma|-a_0}\right), \infty\right)$, provided that $|\gamma| > a_0$. If $a_0 = |\gamma|$ the expansion will be asymptotically linear in $t$, and for $a_0 > |\gamma|$ the expansion will be very accelerated, leading the Universe to the *big rip*. For models with $k = -1$ and $\gamma > 0$, the solution of the Eq. (13) is

$$\tilde{a}(t) = a_0 \cosh(t) + |\gamma| \left(\sinh(t) - t\right). \tag{15}$$

For models with $k = 0$, the solutions of (13) do not depend on the NC parameter, leading to Einstein's static Universe. For models with $k = 1$ and $\gamma < 0$, the solution of (12) has the form

$$\tilde{a}(t) = a_0 \cos(t) + |\gamma| \left(t - \sin(t)\right). \tag{16}$$

It describes a Universe with multiple alternated phases of acceleration and deceleration. We may qualitatively identify the first (accelerated) phase with the inflationary phase of the primordial Universe, followed by a deceleration phase (which allows the agglutination of matter and the formation of structures). The subsequent accelerated expansion phase can be qualitatively associated to the current expansion that is observed today. Such model predicts that this process takes place over and over again, with acceleration phases for $t \in \Big(\theta + 2n\pi,\ \theta + (2n+1)\pi\Big)$, in which $n$ is an integer number and $\theta = arctan\left(\frac{a_0}{|\gamma|}\right)$, and deceleration phases for $t \in \Big(\theta + (2n+1)\pi,\ \theta + 2(n+1)\pi\Big)$. The inflection points of the expansion happen for $t = \theta + m\pi$, in which $m$ is an integer.

A similar picture of two accelerated phases with an in-between decelerated phase has been obtained by Nivaldo *et. al* [4] with the quantization of a plane FRW model with dust and positive cosmological constant. In the present work, the same effect of cosmological repulsion has been obtained with the non-commutative parameter $\gamma$ being responsible for the expansion of the Universe, thus avoiding the *big crunch* predicted by the commutative model. Moreover, the smaller the value of $\gamma$ the larger the Universe acceleration. Therefore, the periodic

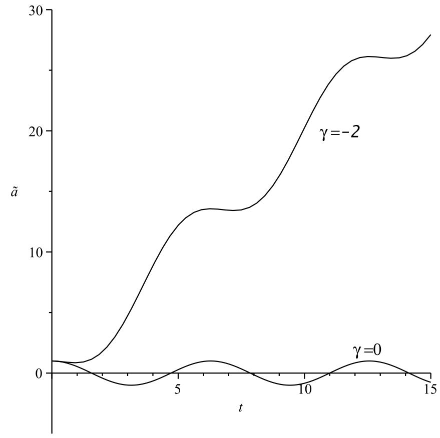


FIG. 1. Evolution of the scale factor $\tilde{a}(t)$ with $k = 1$ for the commutative ($\gamma = 0$) and noncommutative cases ($\gamma < 0$) (here, $\gamma = -2$ has been chosen arbitrarily for the sake of comparison). Observe the elimination of future singularities for the NC case.

acceleration of our Universe is obtained without the introduction of cosmological constant or scalar fields (cf. Fig.1). Hence fitting the results obtained in the NC-FRW model with observational data may require the consideration of some other matter content.

For models with $k = 1$ and $\gamma > 0$, the solution of Eq.(13) has the form

$$\tilde{a}(t) = a_0 \cos(t) - |\gamma| \left(t - \sin(t)\right) , \tag{17}$$

leading to a inevitable collapse, down to the singularity $\tilde{a} = 0$ for some $t > 0$.

## I. ACKNOWLEDGEMENTS

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